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A Natural Golden Spiral

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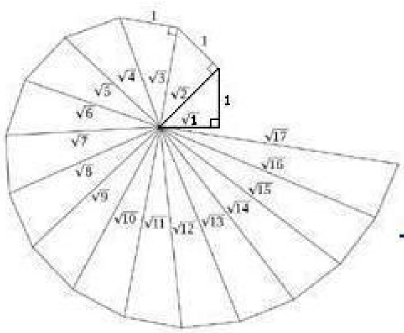
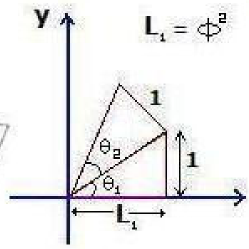
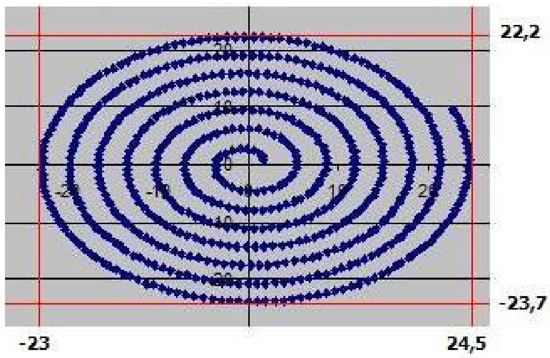
In the previous article I claimed that the Golden Ratio Φ can be considered as the ***elementary creative number*** for many living beings. But in many cases Φ is hidden in the initial formation and cannot be immediately identified. The spiral, known as the Golden Spiral is the one shown in the previous article. It is a logarithmic spiral whose growth factor is Φ.

Evolution and growth is not a linear process, but is discontinuous and iterative. Therefore, the spirals of nature should be modeled using an iterative (self-referential) recursive method. One such example is the **Spiral of Theodorus**. This is formed by adding contiguous right angle triangles, as seen on left below.







Let us apply the same method but with the base of the right triangle equal to $\mathbf{L}_1 = \Phi^2$. The hypotenuse of the first triangle becomes the base of the second triangle, and so on. The base of the n'th triangle is $L_n = (n+\Phi^4)^{1/2}$. The angle Θ_1 of the first triangle can be calculated from $\tan(\Theta_1) = 1/\Phi^2$ and the angle of the n'th triangle from $\tan(\Theta_n) = 1/(n-1+\Phi^4)^{1/2}$. The distance of the n'th point to the origin is the average of two consecutive distances, $R_n = (L_n + L_{n+1})/2$. The distance of any point to the two axes is then:

$$\mathbf{X_n = R_n \cos(\varphi_n)} \text{ and } \mathbf{Y_n = R_n \sin(\varphi_n)}$$
$$\text{with } \boldsymbol{\varphi_n = \Theta_1 + \Theta_2 + \Theta_3 + \ldots\ldots\ldots + \Theta_n}$$

The spiral drawn above right may look like an ellipse, but in fact is more similar to a circle. As the windings increase its shape approaches a circle for large enough n. We find that at the 6th winding the spread on x-axis is 47.5 and on the y-axis 46.9. The average distance between two consecutive windings approaches **π** at the limit. For the above spiral the average distance is given in the Table below.

Winding	Distance-X	Distance-Y
1	3,26959	3,47344
2	3,20510	3,24906
3	3,16510	3,20264
4	3,17452	3,18065
5	3,15793	3,15933
6	3,15515	3,15521
.	.	.
.	.	.
.	.	.
∞	3,14159..	3,14159..

Although one can find many different forms of incorporating the Φ in the equation of a spiral, the present one has the advantage to exhibit a connection to π. The circumference of a circle is $2\pi R$ and the area enclosed within a circle is πR^2. As the length of the major and minor axis equalize, the spiral approaches a circle. The circle being the shape in two dimensions with the largest area for a given length of perimeter, it also conforms to the second law of Thermodynamics. According to this law, a system with many particles tends to occupy the maximum number of available states or positions possible. In other words the Entropy for a close system increases until it reaches a stable state. This is why most stellar objects, such as the sun and the moon, as well as soap bubbles are spherical.

There are several examples in nature that exhibit quasi-circular shapes. Here are some of them. These are microorganisms that are called foraminifers. Most of them are fossilized and do not live any more. It is very possible that they were the very first organisms alive on earth.