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## Paradoxes of Logic

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The bivalent logic, which started with the ancient Greek philosophers, is based on well defined and isolated concepts forming a long chain of closed systems and sets. One starts with a well defined hypothesis A and through a series of steps, B, C, D,... reaches a final conclusion. The problem arises when A leads to B, but B **points back** to A; when there is a recursive interaction between A and B. Then we are stuck and the circular, self referential reasoning leads to a paradox. Philosophers such as Zeno (about 400 BC) and Epimenides (about 600 BC) saw the shortcomings of dichotomy and designed ingenious paradoxes.

### **Achilles and the tortoise paradox**

A race is organized between the fast runner Achilles and the slow tortoise. Achilles, sure of his speed, gave some starting advance to the tortoise. Zeno argued that the fast runner Achilles can never overtake the tortoise, since Achilles must first reach the point whence the tortoise started, so that at every new distance -or step- the slower tortoise must always hold a lead and Achilles can never catch up with the tortoise.

This way of reasoning is correct if only distance is taken into account. Speed, on the other hand, is a function of both distance as well as time. The definition of speed is the ratio of distance over time ( $v = x/t$ ). If for a fixed amount of time the fast runner covers a larger distance, then the speed of the fast runner will exceed the speed of the slow runner and there will be no paradox at all.

But the paradox has deeper implications when carefully analyzed. Achilles has to complete any step in a finite duration of time. Before he completes a step in time  $t$ , he must cover the distance of half a step in time  $t/2$  and this time interval can also be divided in two halves,...ad infinitum. The limit of halving the time intervals leads to an interval of zero duration. This means that time, is in fact, an illusion, and any starting point contains a singularity. We found the same problem when trying to understand the Big Bang (see article **2-Problems of the Big Bang**).

In order to circumvent this difficult problem of singularity, Newton accepted that there exists a limit where the time interval is infinitesimally small, but never becomes equal to zero. Thus, he defined speed as:

**$V = \lim_{\Delta t \rightarrow 0} (\Delta x / \Delta t)$** , where  $\Delta t = t_2 - t_1$  is a finite difference.

Such a way of defining the speed, presupposes that  $\Delta t$  can never be equal to zero. In other words, although very small, a well defined nonzero time interval is **postulated** as a valid physical reality. If both  $\Delta x$  and  $\Delta t$  become equal to zero, then their division becomes indefinite and no finite speed can be defined. So, Zeno's paradox is still valid at the limit of an infinitesimally small time interval. Such an instantaneous time interval is the "**now**" and the infinitesimal distance interval is the "**here**".

Many philosophers and eastern mystics have claimed that the extended time concept is an illusion of the mind and that the only reality is the **now** and the **here**. The same is true for Quantum Theory which claims that reality can only be stated as a function of "here" and "now". This was discussed in article **8-Entangled States**, with the Schrödinger's cat as an example. The cat within the box, with all other items in the box, forms an interacting conglomerate that exists as a single vibrating wave. As soon as we open the door of the box and make an observation, we collapse the wave function and identify the contents of the box as independent, non-interacting entities. Observation is always valid for "here" and "now". Quantum theory does not make any assumption regarding the past or the future of a system. The past and the future are -to a large extent- constructs of the brain. They are linear extrapolations stemming from our method of rationalizing. Any observation is, in fact, a spontaneous symmetry breaking or a bifurcation of the holistic wave-nature of reality.

If we get rid of our pre-suppositions and assumptions we will be able to see the unity and the holistic connectivity of all that is. So, all that is 'out there' is both discrete as a particle AND interconnected as a wave. The external world is both distinct from ourselves AND is part of ourselves. There is a never ending instantaneous interaction between ourselves AND the background we live in. The AND's are in capital letters, because I wanted to stress the new way of thinking developed in the previous article.

The bivalent way of thinking leads to paradoxes, because one assumes that there exist rigorous and precise distinctions among concepts which do not overlap or interact. But in fact the overlapping and interaction is a must for every natural physical set or structure. Dr. Niles Eldredge, a leading scientist on **Punctuated Equilibria** (1) says:

And we must consider the possibility that other sorts of entities -molecular as well as large-scale entities, such as species and ecosystems- also exist and take part in some fashion in the evolutionary process. It is this multi-level view of the organization of nature, a view to which punctuated equilibria has contributed not a little, that is likely to give us an evolutionary theory more in tune with life's actual history on Earth.

These words are in accordance with the fact that all systems, micro as well as macro, interact and do not exist as isolated, independent and closed systems or sets of similar individuals. If we do not adapt our way of thinking to the "multi-level and intertwined" structure of nature, we are bound to always encounter paradoxes and unanswerable problems.

The Austrian logician and mathematician Kurt Gödel (1906-1978) is well known for his incompleteness theorem related to formal logical systems. His theorem came as a blow to the mathematicians who were trying to uncover the foundations of mathematics with the help of set theory. His theorem proved that any formal system build out of "self-consistent" rules will contain unanswerable questions, and therefore cannot be complete. Furthermore, the consistency of the rules cannot be proven within the system. Thus, self-consistency is an undefined concept, which results from assuming that closed sets exist.

This means that no consistent system can be used to prove its own consistency and that it will always be possible to find statement of even elementary logic that can be both proved AND disproved. Here again we are faced with unity of opposites. This unity is a universal truth that holds **even for** any axiomatic set theory.

Gödel's theorem rests on a self-referential equation that can be put in daily language as follows: "this sentence cannot be proved with the set of axioms and rules valid in the present logical system". Since proof means reaching a final conclusion which says either "yes" or "no" in a clear cut manner, then the theorem proves that such a conclusion is impossible. The final answer will contain paradox saying both yes AND no. Therefore, any formal logical system can never be totally closed and complete.

### **The liar's paradox**

A similar paradox known as the liar paradox, was designed by Epimenides of Creta. He made the following statement:

All Cretans are liars. I am a Cretan. Am I lying or not?

Since he is a Cretan, he should be lying, and therefore his statement should be false. This is because the word "all" defines a complete set of identical individuals. According to the definition of a closed set, we cannot say "some Cretans lie and some don't". But if his statement is a lie, then all Cretans are-not liars. Thus, Cretans are both liars AND not-liars. According to the formal bivalent logic this is a paradox. The paradox arises from the **law of excluded middle**.

The law of excluded middle says that any proposition is either true or false. It also says the there are only two possibilities, either all or none. The third possibility being "some" is excluded. But in nature there are many situations which correspond to the middle case. For example, you take a pot of black ink and start adding water in small quantities to the pot. At a given point the ink will become gray, that is, its color will be neither black nor white; or both black AND white in equal proportion. But it will not be possible to tell exactly which drop of water turned the ink to become a "true gray". **Truth**, in nature is fuzzy and very subjective.

(1) **New Scientist Magazine**, 5 June 1986, page 54.