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Conservation Laws and Symmetry

Doç. Dr. Haluk BERKMEN

Emmy Noether

In 1915 the German mathematician **Emmy Noether** (1882 – 1935) proved that the conservation laws are the result of symmetries that exist in nature (1). Time translation symmetry gives conservation of energy; space translation symmetry gives conservation of linear momentum; rotation symmetry gives conservation of angular momentum. But it is also possible to argue the other way around. Because energy is conserved in nature (energy cannot be destroyed) certain features of a system do not change as time goes by. And also because linear momentum is conserved in nature certain features of the system do not change when the system changes place.

E. Noether used the ***Langrangian*** to prove her theorem. The Lagrangian is defined as: **L = T – V**, where T is the kinetic energy and V is the potential energy of a particle. It has been demonstrated that the Lagrangian must satisfy the following equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

The q variable is the generalized coordinate and can be taken as x for a particle moving along a straight line. Thus $\mathbf{q}^{\cdot}$ = dx/dt = v (the speed of the particle). Let us take a simple example where the particle of mass m is moving at constant speed v along a straight line. The Kinetic energy of the particle is $\mathbf{T = mv^2 /2}$ and its Potential energy is **V =0.** Thus $\mathbf{L = mv^2 /2}$ and dL/dq = dL/dx = 0 since the Lagrangian does not depend on the distance x. But it depends on v and dL/dv = mv (the linear momentum of the particle). We get: **d(mv)/dt = 0** and **mv = constant**

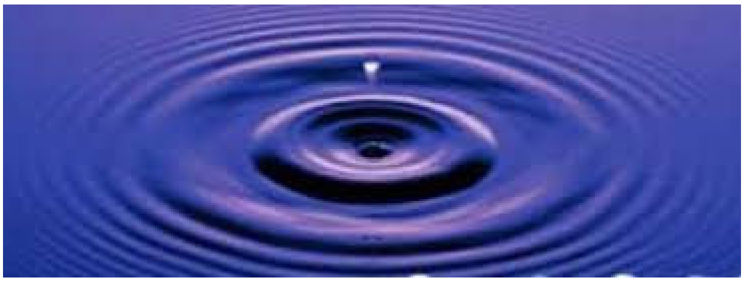
But **d(mv)/dt = m(dv/dt) = ma = F = 0,** where dv/dt = a the acceleration of a particle and according to Newton's law **F = ma** we find that there is no net force acting on the particle F = ma = 0. We also have found that the linear momentum is conserved, since **mv** is **constant**. The linear momentum does not depend on the position x, and we can say that "if the particle's motion is independent of its position its linear momentum is conserved". Or vice versa "If the linear momentum is conserved space translation is valid".

It is also possible to show that if an object's motion has a rotational symmetry the angular momentum is conserved. By "rotational symmetry" I mean that as time goes by the particle returns to the same point that it occupied before. This is the case of all planets revolving around the sun. This is why the seasons come recurring year after year with a predictable regularity. The daily variations are due to the local weather fluctuations.

The conservation of angular momentum is also valid for electrons revolving around the atom's nucleus. **Niels Bohr** (1885 – 1962) has developed his atom model by accepting that electrons move along circular orbits and that their angular momentum is fixed according to $\mathbf{L = nh/2\pi}$. Here h is the Planck constant and n = 1, 2, 3, …. Fixing the angular momentum to a certain constant number is equivalent to say that the angular momentum is conserved and quantized (2). But electrons move also along different *orbitals*, and this knowledge led to add more quantum numbers to the atomic model of N. Bohr (3).

We know that the motion of planets around the sun is either circular or elliptic. But how and why did these orbits appear at the first place? Newton's law of gravitation is about the force acting between stellar objects, but does not tell us anything about the formation of these orbits. Einstein, on the other hand, claimed that stellar objects curve and deform the space around them. Attraction is then explained as a natural motion along this curve, without the need to impose the presence of a gravitational force. Moving on a curved space without the need of an external force is defined as moving along a ***geodesic***. A geodesic is the shortest line connecting two points on a curved surface and is equivalent to a straight line on a flat space.

The Geodesic principle says that particles moving along geodesics do not require an external force. This statement is the equivalent of Newton's first law that claims: "If no force acts on an object, the object is either at rest or is in a state of uniform motion at constant speed". A planet revolving uniformly around the sun in a circular orbit is moving along a geodesic. Because of angular momentum conservation the uniform character of the planetary motion is preserved even if the planet moves along an elliptical orbit.



In order to visualize this situation let us consider the picture on the left. When a heavy sphere (a marble) is dropped on the water, circles are formed on the surface. This is a 2-dimentional phenomenon. The space-time structure of the universe is 4-dimentional. Thus the sun, which is a heavy object in the sky, distorts the 4-dimentional space-time structure in a similar way and creates concentric circles around it. Each circle is a geodesic and planets move along these geodesics without requiring any external force. The difference is that planetary orbits are fixed and do not expand like circles on the water.

References

(1) https://en.wikipedia.org/wiki/Noether%27s_theorem

(2) https://en.wikipedia.org/wiki/Bohr_model

(3) https://en.wikipedia.org/wiki/Atomic_orbital