Subject: Science Article: 14

The Golden Ratio

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In the previous chapters we saw that at the critical point a spontaneous symmetry breaking occurs and the system in question ends up in a new 'stasis'. A stasis is a state of existence that can be, either stable, periodic or even chaotic. I consider the **chaotic state** to be a stasis because it does not diverge and dissipates but keeps a hidden connection to its **Strange Attractor**.

In chapter **11-Order and Disorder** I claimed that there is a fundamental equation that has the power to demonstrate the existence and behavior of many natural structures and phenomena. The general form of this equation is:

$$\mathbf{T}(\mathbf{x}) = \mathbf{\Sigma}_n \mathbf{a}_n \cdot \mathbf{x}^n$$

where \sum_{n} means "sum over many integers n, starting from zero" and the a_n are the order parameters of the state under consideration. Restricting the infinite sum to only three order parameters it is possible to explain the behavior of many natural structures and phenomena. Let us choose $a_0 = a_1 = 1$ and $a_2 = -1$ and all other a_n parameters equal to zero. Then the above equation becomes:

$$T(x) = 1 + x - x^2$$

The simplest case is when T(x) is equal to zero. The two roots of this equation $1 + x - x^2 = 0$ are;

$X_1 = 1.6180339887....$ and $X_2 = -0.6180339887.....$

These two roots are never ending irrational numbers and $X_1 = \Phi$ is known since early times as the **Golden Number** or the **Golden Ratio**. The above equation can now be written as: $\mathbf{1} + \Phi - \Phi^2 = 0$, which gives $\Phi^2 = \mathbf{1} + \Phi$.

The division of any two numbers A and B -where B is larger than A- will be the Golden Ratio Φ if, $B/A = \Phi$. Inserting this ratio into $\Phi^2 = \mathbf{1} + \Phi$ we get a general expression which is valid for all A and B and is independent of Φ :

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(A + B) / B = B / A

Let us now consider two geometrical examples.

The figure at the left starts from a square whose side length is 1. We calculate the side of the next square from the sum of the last two numbers. Thus the sides increase as $F_n = F_{n-1} + F_{n-2}$ and this series is known as the Fibonacci series after Leonardo Fibonacci (1170-1250), the Italian mathematician who brought the Arabic numbers to Europe. If we start with $F_0 = 0$ and $F_1 = 1$, then we get, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$... and so on, as seen above. The ratio between two consecutive Fibonacci numbers F converges to Φ as F becomes large enough. For example F_{30} / $F_{29} = 832040$ / 514229 = 1.618033989.... is equal to Φ up to 8th decimal place.

On the figure at the right we see that the pentagram also contains the Golden Ratio. The flower below has also the Golden Ratio as its initial formation. From the logic developed in the previous chapters we can conclude that Φ becomes an **elementary creative number** for many living beings. Φ being an irrational number has no end. The nautilus, the star and the flower can grow indefinitely and do not end theoretically. The limit is decided by the surroundings.









We see above several live formations that contain the Golden Ratio. All of them grow in spirals. An important characteristic of the spiral is that it is self similar and fractal (see previous article **13-Fractal Adaptation**).

In many plants leaves grow according to the Golden Ratio. The angle that splits a circle according to the Golden Ratio is 137.5 degrees. If each new leaf grows making an angle of 137.5 degrees with its predecessor, the new leaf will not obstruct the sunshine from the leaves below. We see that even after 27 leaves, no leaf obstructs the sunshine of any previous leaf.

The numbers on the black columns indicate in which turn –each turn being 360 degrees- the leaf belongs. The horizontal lines indicate the angle that the leaf makes with the first one. There are no two black columns sharing the same horizontal line. This, in fact is amazing.

